

Deutsch's algorithm (Application of phase kick-back)

$f: \{0,1\} \rightarrow \{0,1\}$, problem: decide whether f is constant $\begin{pmatrix} 00 \\ 11 \end{pmatrix}$ OR balanced $\begin{pmatrix} 01 \\ 10 \end{pmatrix}$
 $f(0) = f(1)$
 OR
 $f(0) \neq f(1)$



$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} [|0\rangle + |1\rangle] \xrightarrow{U_f} \frac{1}{\sqrt{2}} \left[(-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle \right]$$

because $2f(0) = 0 \pmod{2}$

$$= \frac{1}{\sqrt{2}} (-1)^{f(0)} \left[|0\rangle + (-1)^{f(0)+f(1)} |1\rangle \right]$$

$|+\rangle = |0\rangle + |1\rangle$ if $f(0) = f(1)$
 $|-\rangle = |0\rangle - |1\rangle$ otherwise

$\downarrow H$
 $(-1)^{f(0)} H|+\rangle = |0\rangle$
 if $f(0) = f(1)$

$\downarrow H$
 $(-1)^{f(0)} H|-\rangle = (-1)^{f(0)} |1\rangle$
 if $f(0) \neq f(1)$