Deutsch-Jozsa Algorithm (Application of Fourier sampling)

Promise: f is constant OR f is balanced

Problem: tell which case

Assume f is constant : $f(x) = c \quad \forall (x)$

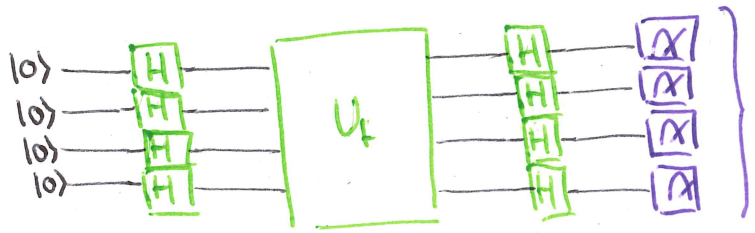
$$\longrightarrow \hat{\Phi}(y) = \frac{1}{2^n} \sum_x (-1)^{c + xy} = (-1)^c \frac{1}{2^n} \sum_x (-1)^{xy} = \begin{cases} (-1)^c & \text{if } y = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow |\hat{\Phi}(y)|^2 = 1$$

$$\Rightarrow |\hat{\Phi}(y)|^2 = 0$$

Assume f is balanced

$$\longrightarrow \hat{\Phi}(0) = \frac{1}{2^n} \sum_x (-1)^{f(x) + x \cdot 0} = \frac{1}{2^n} \sum_x (-1)^{f(x)} = 0$$



obtain y with prob. $|\hat{\Phi}(y)|^2$

- 0 $\quad y \neq 0$
- 1 $\quad$ if $f(x) = c$
- 1 $\quad$ otherwise

- 1 query to check whether f is constant or balanced
- Classical: $\lfloor \frac{n}{2} \rfloor + 1$ queries to decide with prob. 1

BUT Bounded-error Probabilistic Polynomial-time (BPP)

- correct answer with prob $\geq \frac{2}{3}$ (possible to end after 2 queries)
  $f(x) = 0$, $f(y) = 1 \rightarrow$ STOP